MATH 201C SPRING 2014 PRACTICE FINAL

Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your five best answers.

(1) Let A, B, and C be subsets of a set U. Prove that
$$(C \smallsetminus A) \cup (A \smallsetminus B) \cup (B \smallsetminus C) = (A \cup B \cup C) \smallsetminus (A \cap B \cap C)\,.$$

(2) Let n be an integer. Prove that $5 \mid (n^4 - 1)$ implies $5 \nmid n$.

(3) Show that $\log_3 \sqrt{5}$ is irrational.

(4) Let n be a positive integer. Prove or disprove:
$$3 \mid (2^{2n} - 1)\,.$$

(5) Show that the series
$$\sum_{n=2}^{\infty} \frac{1}{n \log n}$$
diverges to infinity.

(6) (a) Explain why the function $f(x) = x$ is continuous at $x = 10$.
(b) Use limit theorems and induction on n to show that
$$x^n + x^{n-1} + \ldots + x^2 + x$$
is continuous at $x = 10$ for each positive integer n.