MATH 2010 FALL 2026 PRACTICE TEST #2

Write clearly, on separate paper.

(1) [5pts.] (a) Show that the set $E = \{q \in \mathbb{Q} \mid 2^q < 3\}$ is bounded above in $(\mathbb{Q}, <)$.

(b) Show that E does not have a least upper bound in $(\mathbb{Q}, <)$.

(2) [5pts.] Consider an element x of an ordered field. If $-1 \leq x$, show that

$$1 + nx \leq (1 + x)^n$$

for all natural numbers n.

(3) [4pts.] Prove that the function

$$[-2, 1] \to \mathbb{R}; x \mapsto x^4 + 3x^2 - 7\cos x$$

is bounded, using the methods available from our class.