MATH 2010 FALL 2026 GRADED HOMEWORK #2

Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your three best answers.

(1) Consider the functions $f, g\colon \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = x^3 \quad \text{and } g(x) = \sqrt{x^2 + 1}\,.$$

Find the formulas for $f \circ g$ and $g \circ f$.

(2) Provide a one-line proof to find the inverse of the function $f\colon \mathbb{R} \smallsetminus \{1/2\} \to \mathbb{R} \smallsetminus \{5/2\}$ with

$$f(x) = \frac{5x - 2}{2x - 1}\,.$$

(3) Prove by induction:

Proposition. For each natural number n, if each member of $\{A_1, A_2, \ldots, A_n\}$ is a countable set, then $A_1 \cup A_2 \cup \ldots \cup A_n$ is countable.

(4) Prove by induction: $\forall\ 3 < n \in \mathbb{N}\,,\ 2n + n^2 \leq n!$.