

MATH 2010 FALL 2025 GRADED HOMEWORK #2

*Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your three best answers.*

- (1) Consider the functions $f, g: [0, \infty[\rightarrow [0, \infty[$ defined by

$$f(x) = \sqrt{x^2 + 1} \quad \text{and} \quad g(x) = x^2.$$

Find the formulas for $f \circ g$ and $g \circ f$.

- (2) Find the inverse $g(y)$ of the function $f: \mathbb{R} \setminus \{1\} \rightarrow \mathbb{R} \setminus \{3\}$ with

$$f(x) = \frac{3x - 2}{x - 1}.$$

- (3) Consider the set

$$\mathbb{Z}[\sqrt{2}] = \{m + n\sqrt{2} \mid m, n \in \mathbb{Z}\}.$$

Prove that $\mathbb{Z}[\sqrt{2}]$ is countably infinite.

- (4) Prove by induction: $\forall 5 \leq n \in \mathbb{N}, 2n^2 \leq n!$.