MATH 2010-2 FALL 2024 GRADED HOMEWORK #3

Write clearly, on separate paper. All questions carry equal weight.

(1) Determine the limit of the sequence

$$\left\{ \frac{2n^3 + 5n^2 \cos(3n)}{5n^3 - n + 1} \right\}_{n \in \mathbb{N}} ,$$

justifying your answer.

(2) Give a careful proof, by induction, of the following

Proposition. For each natural integer n, the function $(1-x)^n$ is continuous.

(3) Let $\{x_n\}_{n \in U}$ be a sequence such that

$$\forall\ m, n \in U\ ,\ |x_m - x_n| < \frac{1}{m} + \frac{1}{n}\,.$$

Give a careful, direct proof that $\{x_n\}_{n \in U}$ is a Cauchy sequence.