MATH 2010-2 FALL 2024 GRADED HOMEWORK #2

Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your three best answers.

(1) Consider the functions $f, g\colon \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \sqrt{x^2+1} \quad \text{and } g(x) = x^3\,.$$

Find the formulas for $f \circ g$ and $g \circ f$.

(2) Find the inverse of the function $f\colon \mathbb{R} \smallsetminus \{2\} \to \mathbb{R} \smallsetminus \{2\}$ with

$$f(x) = \frac{2x-3}{x-2}\,.$$

(3) Prove or disprove:

Proposition. For sets A and B, if $A \smallsetminus B$ and A are countable, then B is countable.

(4) Prove by induction: $\forall\ 4 < n \in \mathbb{N}\,,\ n + n^2 < 2^n$.