

MATH 201 FALL 2023 PRACTICE FINAL

Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your five best answers.

- (1) Prove or disprove:

Let A , B , and C be subsets of a set U .

Then $(C \setminus A) \cup (A \setminus B) = (A \cup C) \setminus B$.

- (2) Prove or disprove:

The inequality $4x^2 + 25y^2 \geq 20xy$ holds
for all real numbers x and y .

- (3) Prove or disprove:

For real numbers x, y with $x + y > -1$,

$$(1 + x + y)^n \geq 1 + nx + ny$$

for each natural number n .

- (4) Prove or disprove:

The function

$$f: \mathbb{Q} \times \mathbb{Q} \rightarrow \mathbb{R}; (p, q) \mapsto \frac{(1 - p + pq)(1 + q - pq)}{1 + p^2 + q^2}$$

is invertible.

- (5) Prove or disprove:

Suppose that $\{x_n\}$ and $\{y_n\}$ are convergent sequences
of real numbers with $x_n \leq y_n$ for all n .

Then $\lim_{n \rightarrow \infty} x_n \leq \lim_{n \rightarrow \infty} y_n$.

- (6) Prove or disprove:

The series

$$\sum_{n=2}^{\infty} \frac{1}{n \log n}$$

diverges.