MATH 201 FALL 2023 PRACTICE TEST #2

Write clearly, on separate paper.

(1) [5pts.] Let A be a totally ordered set with the least upper bound property. Let E be a nonempty subset that is bounded below. Let L be the set of lower bounds for E.
   (a) Show that L is nonempty.
   (b) Show that L is bounded above.
   (c) Show that $\sup L$ is a lower bound for E.
   (d) Show that $\sup L$ is the greatest lower bound of E.

(2) [5pts.] Show that the set

$$\left\{ \frac{m}{n} \in \mathbb{Q} \,\middle|\, 2^m < 3^n \text{ and } 0 < m, n \in \mathbb{N} \right\}$$

does not have a least upper bound in $(\mathbb{Q}, <)$. You may assume that the real number $\log_2 3$ is irrational.

(3) [4pts.] Find

$$\sup \left\{ \frac{5n^2 - 3n}{2n^2} \,\middle|\, 0 < n \in \mathbb{N} \right\}.$$