

MATH 201 FALL 2023 GRADED HOMEWORK #3

Write clearly, on separate paper. All questions carry equal weight.

- (1) Determine the limit of the sequence

$$\left\{ \frac{n^2 - 4n \cos 4n}{7n^2 - 2} \right\}_{n \in \mathbb{N}},$$

justifying your answer by quoting known propositions.

- (2) For each natural number m , suppose that the sequence $\{a_{mn}\}_{n \in U}$ is convergent. Prove by induction that the sequence

$$\{a_{0n} + a_{1n} + \dots + a_{rn}\}_{n \in U}$$

is convergent for each $r \in \mathbb{N}$.

- (3) Let $\{a_n\}_{n \in U}$ and $\{b_n\}_{n \in U}$ be convergent sequences, with

$$\lim_{n \rightarrow \infty} a_n \neq \lim_{n \rightarrow \infty} b_n.$$

Show that there is a natural number M such that $a_n \neq b_n$ for all $M \leq n \in U$.