

MATH 201 FALL 2023 GRADED HOMEWORK #2

Write clearly, on separate paper. All questions carry equal weight.

- (1) Find the inverse of the function $f: \mathbb{R} \setminus \{\frac{9}{8}\} \rightarrow \mathbb{R} \setminus \{\frac{1}{2}\}$ with

$$f(x) = \sqrt[3]{\frac{x-1}{8x-9}}.$$

- (2) Prove the following result by induction:

Proposition. Let $\{A_r \mid r \in \mathbb{N}\}$ be a family of countable sets. Then for all natural numbers n , the set

$$A_0 \cup A_1 \cup \cdots \cup A_{n-1} \cup A_n$$

is countable.

- (3) Prove by induction: $\forall 6 \leq n \in \mathbb{N}, 2^n \leq (n-1)!$.