MATH 201 FALL 2022 PRACTICE TEST #2

Write clearly, on separate paper.

(1) [5pts.] For real numbers x, y, suppose $y - x > 1$. Prove:

$$\exists\, n \in \mathbb{Z}\,.\; n \in (x, y)\,.$$

(2) [5pts.] Consider a real number x with $|x| < 1$. Prove that

$$|1 - x + x^2 - x^3 + \cdots + (-1)^r x^r| \leq \frac{1 - |x|^{r+1}}{1 - |x|}$$

for $r \in \mathbb{N}$.

(3) [4pts.] Suppose that R is a positive real number. Consider a polynomial $p\colon [-R, R] \to \mathbb{R}$ with

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0$$

for $a_n, \ldots, a_0 \in \mathbb{R}$. Show that $|p(x)|$ is bounded by

$$|a_n| \cdot R^n + |a_{n-1}| \cdot R^{n-1} + \cdots + |a_1| \cdot R + |a_0|$$

for $x \in [-R, R]$.