MATH 201 FALL 2022 GRADED HOMEWORK #2

Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your three best answers.

(1) Consider the functions $f, g \colon [0, \infty[\to [0, \infty[$ defined by
$$f(x) = \sqrt[3]{x^2 + 1} \quad \text{and} \quad g(x) = x^3 \,.$$
Find the formulas for $f \circ g$ and $g \circ f$.

(2) Find the inverse of the function $f \colon \mathbb{R} \smallsetminus \{1\} \to \mathbb{R} \smallsetminus \{1\}$ with
$$f(x) = \frac{x - 2}{x - 1} \,.$$

(3) Consider the set
$$\mathbb{Z}[\sqrt{5}] = \{m + n\sqrt{5} \mid m, n \in \mathbb{Z}\} \,.$$
Prove that $\mathbb{Z}[\sqrt{5}]$ is countably infinite.

(4) Prove by induction: $\forall\, 4 \le n \in \mathbb{N}, \ n^2 \le n!$.