MATH 201 FALL 2021 PRACTICE FINAL

Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your five best answers.

(1) Prove or disprove:
Let A, B, and C be subsets of a set U.
Then $(C \cup A) \setminus (A \cup B) = C \setminus B$.

(2) Prove or disprove:
The inequality $x^2 + 9y^2 \geq 6xy$ holds
for all real numbers x and y.

(3) Prove or disprove:
For each prime number p,
the integer $p^2 + p + 1$ is prime.

(4) Prove or disprove:
For each positive integer n,
the integer $n^5 + 5n^3 - n$ is a multiple of 5.

(5) Prove or disprove:
The function

$$f : \mathbb{Q} \times \mathbb{Q} \to \mathbb{R}; (x, y) \mapsto x^2 - 2\pi y - e^x$$

is invertible.

(6) Prove or disprove:
Suppose that $\{x_n\}$ and $\{y_n\}$ are convergent sequences
of real numbers with $x_n < y_n$ for all n.
Then $\lim_{n\to\infty} x_n < \lim_{n\to\infty} y_n$.

(7) Prove or disprove:
The series

$$\sum_{n=1}^{\infty} \frac{3n+2}{n^3}$$

converges.