

MATH 201 FALL 2021 PRACTICE TEST #2

Write clearly, on separate paper.

- (1) [5pts.] For real numbers x, y , suppose $y - x > 1$. Prove:

$$\exists n \in \mathbb{Z}. n \in (x, y).$$

- (2) [5pts.] Consider a real number x with $|x| < 1$. Prove that

$$|1 + x + \cdots + x^r| \leq \frac{1 - |x|^{r+1}}{1 - |x|}$$

for $r \in \mathbb{N}$.

- (3) [4pts.] Consider $f: [0, 1] \rightarrow \mathbb{R}$ with

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x^1 + a_0$$

for $a_n, \dots, a_0 \in \mathbb{R}$. Show that

$$|f(x)| \leq |a_n| + |a_{n-1}| + \cdots + |a_1| + |a_0|$$

for $x \in [0, 1]$.