

MATH 201 FALL 2021 GRADED HOMEWORK #2

*Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your three best answers.*

- (1) Consider the functions $f, g: [0, \infty[\rightarrow [0, \infty[$ defined by

$$f(x) = \sqrt{x^2 + 1} \quad \text{and} \quad g(x) = x^2.$$

Find the formulas for $f \circ g$ and $g \circ f$.

- (2) Find the inverse of the function $f: \mathbb{R} \setminus \{2\} \rightarrow \mathbb{R} \setminus \{1\}$ with

$$f(x) = \frac{x - 1}{x - 2}.$$

- (3) Consider the set

$$\mathbb{Q}[\sqrt{2}] = \{q_1 + q_2\sqrt{2} \mid q_1, q_2 \in \mathbb{Q}\}.$$

Prove that $\mathbb{Q}[\sqrt{2}]$ is countably infinite.

- (4) Prove by induction: $\forall 4 \leq n \in \mathbb{N}, n^2 \leq 2^n$.