MATH 201 FALL 2020 GRADED HOMEWORK #3

Write clearly, on separate paper. All questions carry equal weight.

(1) Give a careful formal proof of the following:

> **Prop.** Suppose that a sequence $\{x_n\}_{n\in\mathbb{N}}$ of nonzero real numbers converges to a nonzero limit x. Then there is a positive real number R such that $|x_n| \geq R$ for all natural numbers n.

(2) Give a formal proof by induction of the following:

> **Prop.** Let $f_i\colon \mathbb{R} \to \mathbb{R}$ be a continuous function for each positive integer i. Then the function
>
> $$f_1(x) \cdot \ldots \cdot f_n(x)$$
>
> is continuous for each positive integer n.

You may quote the following lemma without proving it:

> **Lemma.** If $f\colon \mathbb{R} \to \mathbb{R}$ and $g\colon \mathbb{R} \to \mathbb{R}$ are continuous functions, then $f(x) \cdot g(x)$ is continuous.

(3) Prove that the sequence

$$\left\{\frac{n\cos(2n^2-3)\sin(n^3+1)}{n^2+4}\right\}_{0<n\in\mathbb{N}}$$

converges to zero.