

MATH 201 FALL 2020 GRADED HOMEWORK #2

*Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your three best answers.*

- (1) Consider the functions $f, g: [0, \infty[\rightarrow [0, \infty[$ defined by

$$f(x) = \sqrt{x+1} \quad \text{and} \quad g(x) = x^2.$$

Find the formulas for $f \circ g$ and $g \circ f$.

- (2) Find the inverse of the function $f: \mathbb{R} \setminus \{1/2\} \rightarrow \mathbb{R} \setminus \{5/2\}$ with

$$f(x) = \frac{5x-2}{2x-1}.$$

- (3) Prove the following result by induction:

Proposition. Let $\{A_i \mid i \in \mathbb{N}\}$ be a family of countable sets. Then for all natural numbers n , the set

$$\bigcup_{i=0}^n A_i$$

is countable.

- (4) Prove by induction: $\forall n \in \mathbb{N}, n^2 < 3^n$.