MATH 201 FALL 2018 PRACTICE FINAL

Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your five best answers.

(1) Let A, B, and C be subsets of a set U. Prove or disprove:

$$(C \smallsetminus A) \cup (A \smallsetminus B) = (A \cup C) \smallsetminus B\,.$$

(2) For a real number $x > -1$ and a positive integer n, prove that $(1+x)^n \geq 1 + nx$.

(3) Prove or disprove:
For each prime number p,
the integer $p^2 + p + 1$ is prime.

(4) Prove or disprove:
For each positive integer n,
the integer $n^5 - n$ is a multiple of 5.

(5) Prove or disprove:
If a sequence $\{x_n\}$ is divergent,
then none of its subsequences converge.

(6) Show that the series

$$\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$$

converges, and find the limit.