MATH 201 FALL 2018 GRADED HOMEWORK #3

Write clearly, on separate paper. All questions carry equal weight.

(1) Prove that $(1+x)^n \geq 1+nx$ for positive real numbers x and for natural numbers n. Then show that the sequence $\{p^n\}_{n\in\mathbb{N}}$ is unbounded if $1 < p \in \mathbb{R}$.

(2) Give a proof, by induction, of the following

Proposition. For each natural number n, the function

$$x^n + x^{n-1} + \ldots + x + 1$$

is continuous.

(3) Let $\{x_n\}_{n\in U}$ be a Cauchy sequence. Give a direct proof that if a subsequence $\{x_n\}_{n\in S}$ has a limit L, then the Cauchy sequence $\{x_n\}_{n\in U}$ has L as a limit. Do not assume that general Cauchy sequences are convergent.