

MATH 201 FALL 2017 PRACTICE FINAL

*Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your five best answers.*

- (1) Let A , B , and C be subsets of a set U . Prove or disprove:
$$(C \setminus A) \cap (A \setminus B) = (A \cup C) \setminus (A \cap B).$$
- (2) For a real number x and a positive real number ε , prove that $x^2 + 16 \geq 8x + \varepsilon^2$ implies $|x - 4| \geq \varepsilon$.
- (3) Prove or disprove:
For each positive integer n ,
the inequality $2 \cdot 3^n \leq (n + 2)!$ holds.
- (4) Prove or disprove:
For each positive integer n ,
the integer $n^2 + 43n + 43$ is prime.
- (5) Suppose that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences. Give a careful, direct proof that $\{x_n y_n\}$ is a Cauchy sequence.
- (6) Show that the series

$$\sum_{n=2}^{\infty} \frac{1}{n \log n}$$

does not converge.