

## MATH 201C FALL 2016 PRACTICE FINAL

*Write clearly, on separate paper. All questions carry equal weight.  
You will receive credit for your five best answers.*

- (1) Let  $A$ ,  $B$ , and  $C$  be subsets of a set  $U$ . Prove or disprove:

$$(C \setminus A) \cap (A \setminus B) = (A \cup C) \setminus (A \cap B).$$

- (2) For a real number  $x$  and a positive real number  $\varepsilon$ , prove that  $x^2 + 9 \geq 6x + \varepsilon^2$  implies  $|x - 3| \geq \varepsilon$ .

- (3) Let  $n$  be a positive integer. Prove or disprove:

$$8 \mid (9^n - 1).$$

- (4) Let  $n$  be an integer. Prove or disprove:

$$7 \mid (n^6 - 1).$$

- (5) Suppose that  $\{x_n\}$  and  $\{y_n\}$  are Cauchy sequences. Give a careful, direct proof that  $\{x_n y_n\}$  is a Cauchy sequence.

- (6) Show that the series

$$\sum_{n=2}^{\infty} \frac{1}{n \log n}$$

does not converge.