MATH 201C FALL 2016 GRADED HOMEWORK #3

Write clearly, on separate paper. All questions carry equal weight.

(1) Suppose $K > 1$, say $K = 1 + k$ with $k > 0$.
   (a) Show that $K^n = (1+k)^n > 1+nk$ for each positive integer $n > 1$.
   (b) Show that $\forall\ M > 0\,,\ \exists\ 0 < n \in \mathbb{Z}\,.\ K^n > M$.
   (c) Show that the sequence $\{K^n\}$ diverges.

(2) Consider the function $f\colon \mathbb{R} \to \mathbb{R}$ with

$$f(x) = \begin{cases} 5 & \text{if } x \text{ is rational;} \\ x^2 & \text{if } x \text{ is irrational.} \end{cases}$$

Show that $f(x)$ is not continuous at $x = 0$.

(3) Let $\{x_n\}$ and $\{y_n\}$ be Cauchy sequences. Give a direct proof that $\{x_n - 2y_n\}$ is a Cauchy sequence.