

## MATH 201C FALL 2016 GRADED HOMEWORK #2

*Write clearly, on separate paper. All questions carry equal weight.  
You will receive credit for your three best answers.*

- (1) Consider the functions  $f, g: \mathbb{R} \rightarrow \mathbb{R}$  defined by

$$f(x) = \sqrt{x^4 + 1} \quad \text{and} \quad g(x) = x^4.$$

Find the formulas for  $f \circ g$  and  $g \circ f$ .

- (2) Find the inverse of the function  $f: \mathbb{R} \setminus \{-\frac{1}{2}\} \rightarrow \mathbb{R} \setminus \{\frac{3}{2}\}$  with

$$f(x) = \frac{3x + 2}{2x + 1}.$$

- (3) Prove or disprove the following:

**Claim.** Suppose that  $A_i$  is an uncountable set for each real number  $i$ . Then  $\bigcap_{i \in \mathbb{R}} A_i$  is uncountable.

- (4) Prove or disprove the following:

**Claim.** Suppose that  $X$  is a countably infinite set, and  $Y$  is an uncountable set. Then if  $X \subset Z \subseteq Y$  (so  $X$  properly contained in  $Z$ ), the set  $Z$  is uncountable.