

MATH 201C FALL 2015 PRACTICE FINAL

Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your five best answers.

- (1) Use a truth table to decide whether or not the following statements (a) and (b) are logically equivalent:

$$(a) \quad (\sim P) \wedge (P \vee Q) \qquad (b) \quad \sim (Q \rightarrow P).$$

- (2) Let A , B , and C be subsets of a set U . Prove that
 $(C \setminus A) \cup (A \setminus B) \cup (B \setminus C) = (A \cup B \cup C) \setminus (A \cap B \cap C)$.

- (3) Let n be a positive integer. Prove or disprove:

$$5 \mid (4^{2n} - 1).$$

- (4) Use induction to prove that for each $n \in \mathbb{N}$,

$$1 + x + x^2 + \dots + x^{n-1} + x^n = \frac{x^{n+1} - 1}{x - 1}$$

for any real number $x \neq 1$.

- (5) Show that the series

$$\sum_{n=2}^{\infty} \frac{1}{n \log n}$$

diverges to infinity.

- (6) (a) Explain why the function $f(x) = x$ is continuous at $x = 10$.
(b) Use limit theorems and induction on n to show that

$$x^n + x^{n-1} + \dots + x^2 + x$$

is continuous at $x = 10$ for each positive integer n .