MATH 201C FALL 2015 PRACTICE TEST #2

Write clearly, on separate paper.

(1) [4pts.] Consider a real number x with $|x| < 1$. Show that

$$|1 + x + x^2| \leq \frac{1 - |x|^3}{1 - |x|}\,.$$

(2) [4pts.] For real numbers x, y, suppose $y - x > 1$. Prove:

$$\exists\, n \in \mathbb{Z}\,.\; n \in (x, y)\,.$$

(3) [3pts.] Find a number M such that $|2x^3 - 10x^2 - 4| \leq M$ for all $-1 \leq x \leq 3$. Justify your answer.

(4) [3pts.] Give a careful proof that

$$\lim_{n\to\infty} \left(\frac{-1}{n}\right)^3 = 0\,.$$