

### MATH 201C FALL 2015 GRADED HOMEWORK #3

*Write clearly, on separate paper. All questions carry equal weight.  
For full credit, answer all three questions.*

- (1) Let  $x$  be a real number. Consider  $E = \{n \in \mathbb{Z} \mid x < n\}$ . Prove the following statement:

$$\exists M \in E. \forall n \in E, M \leq n.$$

- (2) Let  $\{x_n\}$  and  $\{y_n\}$  be Cauchy sequences. Give a direct proof that  $\{2x_n - y_n\}$  is a Cauchy sequence.

- (3) Consider the function  $f: \mathbb{R} \rightarrow \mathbb{R}$  with

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational;} \\ 1 & \text{if } x \text{ is irrational.} \end{cases}$$

- (a) Show that  $f(x)$  is continuous at  $x = 1$ .  
(b) Show that  $f(x)$  is not continuous at  $x = 0$ .