MATH 201C FALL 2015 GRADED HOMEWORK #2

Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your three best answers.

(1) Show that the real line $\mathbb{R}$ and the open interval (π, ∞) are isomorphic sets.

(2) Find the inverse of the function $f\colon \left(\mathbb{R} \smallsetminus \{-\frac{3}{4}\}\right) \to \left(\mathbb{R} \smallsetminus \{\frac{5}{4}\}\right)$ with

$$f(x) = \frac{5x+4}{4x+3}\,.$$

(3) Give a careful proof of the following:

Proposition. Let A be an uncountable set. Let B and C be subsets of A such that $A = B \cup C$. Then at least one of B, C is uncountable.

(4) Prove or disprove the following:

Claim. Suppose that X and Y are countably infinite sets. Then if $X \subseteq Z \subseteq Y$, the set Z is countably infinite.