MATH 201A FALL 2014 PRACTICE FINAL

Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your five best answers.

(1) Let A, B, and C be subsets of a set U. Prove or disprove:

$(C \smallsetminus A) \cup (A \smallsetminus B) \cup (B \smallsetminus C) = (A \cup B \cup C) \smallsetminus (A \cap B \cap C)$.

(2) Let n be an integer. Prove that $5 \mid (n^4 - 1)$ implies $5 \nmid n$.

(3) Prove or disprove:

$\forall \ 1 < n \in \mathbb{N}, \ \forall \ 1 < d \in \mathbb{N}, \ d \mid (2^n - 1) \Rightarrow d = 2^n - 1$.

(4) For real p, show that

$$p^3 + (1-p)^3 \geq \frac{1}{4}$$

with equality if and only if $p = \frac{1}{2}$.

(5) Give an ε–δ type proof that the real function $x^3 - 2x^2$ is continuous at $x = 3$.

(6) Show that the series

$$\sum_{n=2}^{\infty} \frac{1}{n \log n}$$

does not converge.