

MATH 201A FALL 2014 PRACTICE TEST #2

Write clearly, on separate paper.

- (1) [3pts.] For positive real numbers x, y , show that

$$\sqrt{xy} \leq \frac{x+y}{2}.$$

- (2) [5pts.] For real numbers x, y , suppose $y - x > 1$. Prove:

$$\exists n \in \mathbb{Z}. n \in (x, y).$$

- (3) [3pts.] Find a number M such that $|x^3 - 5x^2 + 2x| \leq M$ for all $-3 \leq x \leq 1$.

- (4) [3pts.] For a sequence $\{x_n\}$, suppose that the subsequences $\{x_{2n}\}$ and $\{x_{2n+1}\}$ are convergent. Show that $\{x_n\}$ is convergent if and only if $\lim_{n \rightarrow \infty} x_{2n} = \lim_{n \rightarrow \infty} x_{2n+1}$.