MATH 201A FALL 2014 GRADED HOMEWORK #3

Write clearly, on separate paper. All questions carry equal weight.
For full credit, answer all three questions.

(1) Consider $R = 1 + r$ with $r > 0$.
   (a) Show that $\forall\ n \in \mathbb{N}\,,\ R^n \geq 1 + nr$.
   (b) Use the result from (a) and the Archimedean Principle to show that the sequence $\{R^n\}_{n\in\mathbb{N}}$ is unbounded.

(2) Let $\{x_n\}$ and $\{y_n\}$ be Cauchy sequences. Give a direct proof that $\{3x_n - 4y_n\}$ is a Cauchy sequence.

(3) Consider the function $f\colon \mathbb{R} \to \mathbb{R}$ with

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational;} \\ \frac{1}{x} & \text{if } x \text{ is irrational.} \end{cases}$$

   (a) Show that $f(x)$ is continuous at $x = 1$.
   (b) Show that $f(x)$ is not continuous at $x = 0$.