

MATH 201B FALL 2013 PRACTICE FINAL

*Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your five best answers.*

- (1) Let A , B , and C be subsets of a set U . Prove or disprove:
 $(C \setminus A) \cup (A \setminus B) \cup (B \setminus C) = (A \cup B \cup C) \setminus (A \cap B \cap C)$.
- (2) Let n be an integer. Prove that $5 \mid (n^4 - 1)$ implies $5 \nmid n$.
- (3) Show that $\log_3 \sqrt{5}$ is irrational.
- (4) Let n be an even integer. Prove or disprove:
 $3 \mid (2^n - 1)$.
- (5) Suppose that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences. Give a careful, direct proof that $\{x_n y_n\}$ is a Cauchy sequence.
- (6) Show that the series

$$\sum_{n=2}^{\infty} \frac{1}{n \log n}$$

does not converge.