

MATH 201B FALL 2013 GRADED HOMEWORK #3

*Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your three best answers.*

- (1) Suppose $K > 1$, say $K = 1 + k$ with $k > 0$.
- (a) Show that $K^n = (1 + k)^n > 1 + nk$ for each positive integer $n > 1$.
 - (b) Show that $\forall M > 0, \exists 0 < n \in \mathbb{Z}. K^n > M$.

- (2) For

$$x_n = \frac{1 + (-n)^3}{n^3},$$

find

$$\liminf_{n \rightarrow \infty} x_n \quad \text{and} \quad \limsup_{n \rightarrow \infty} x_n.$$

- (3) Let $\{x_n\}$ and $\{y_n\}$ be Cauchy sequences. Give a direct proof that $\{x_n - y_n\}$ is a Cauchy sequence.
- (4) (a) For each real number $x \neq 1$ and natural number n , prove

$$1 + x + x^2 + \cdots + x^n = \frac{1 - x^{n+1}}{1 - x}$$

by induction on n .

- (b) Compute

$$\sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k.$$