

MATH 201B FALL 2013 GRADED HOMEWORK #2

*Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your three best answers.*

- (1) Consider the functions $f, g: \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = \sqrt[3]{x-1} \quad \text{and} \quad g(x) = x^2.$$

Find the formulas for $f \circ g$ and $g \circ f$.

- (2) Find the inverse of the function $f: \mathbb{R} \setminus \{\frac{1}{2}\} \rightarrow \mathbb{R} \setminus \{\frac{3}{2}\}$ with

$$f(x) = \frac{3x+2}{2x-1}.$$

- (3) Give a proof by induction of the following:

Theorem. For $1 \leq n \in \mathbb{Z}$, suppose that A_1, A_2, \dots, A_n are countably infinite sets. Then

$$\bigcup_{i=1}^n A_k$$

is countably infinite.

- (4) Prove or disprove the following:

Claim. Suppose that X and Y are countably infinite sets. Then if $X \subseteq Z \subseteq Y$, the set Z is countably infinite.