

MATH 201A FALL 2010 PRACTICE FINAL

*Write clearly, on separate paper. All questions carry equal weight.
You will receive credit for your five best answers.*

(1) Let A , B , and C be subsets of a set U . Prove or disprove:
 $(C \setminus A) \cup (A \setminus B) \cup (B \setminus C) = (A \cup B \cup C) \setminus (A \cap B \cap C)$.

(2) Let n be an integer. Prove that $5 \mid (n^4 - 1)$ implies $5 \nmid n$.

(3) Show that $\log_3 \sqrt{5}$ is irrational.

(4) Let n be an even integer. Prove or disprove:

$$3 \mid (2^n - 1).$$

(5) Give an ε - δ type proof that

$$\lim_{x \rightarrow 3} x^3 - 2x^2 + 1 = 10.$$

(6) Show that the series

$$\sum_{n=2}^{\infty} \frac{1}{n \log n}$$

diverges to infinity.