

## MATH 201A FALL 2010 PRACTICE TEST #1

*Write clearly, on separate paper. All questions carry equal weight.  
You will receive credit for your three best answers.*

- (1) Prove that the statements  $P \leftrightarrow Q$  and  $(P \wedge Q) \vee [(\sim P) \wedge (\sim Q)]$  are logically equivalent.
- (2) Let  $x$  and  $y$  be integers. If  $5x - y$  is odd, prove that  $x$  and  $y$  have opposite parity.
- (3) For an odd integer  $n$ , prove that  $4 \mid n^3 + n^2 + n + 1$ .
- (4) Let  $x$  and  $y$  be positive real numbers. Prove that

$$\frac{x}{y} + \frac{y}{x} \geq 2.$$